

My academic goal for this unit is...

Check for Understanding Key:

- Understanding at start of the unit
- | Understanding after practice
- ▲ Understanding before unit test

LEARNING TARGETS		How is my understanding?	Test Score	Retake?
6a	I can apply properties of chords to determine unknown angle measures and segment lengths.	_____ 1 2 3 4		
6b	I can apply properties of tangents to determine unknown angle measures and segment lengths.	_____ 1 2 3 4		
6c	I can apply the properties of arcs and angles to determine unknown arc and angle measures.	_____ 1 2 3 4		
6d	I can prove circle conjectures.	_____ 1 2 3 4		
6e	I can describe the circumference/diameter ratio and use it to solve problems.	_____ 1 2 3 4		
6f	I can calculate arc length.	_____ 1 2 3 4		

Where does π come from?

What ratio is π ?

DP/1 Developing Proficiency Not yet, Insufficient	CP/2 Close to Proficient Yes, but..., Minimal	PR/3 Proficient Yes, Satisfactory	HP/4 Highly Proficient WOW, Excellent
I can't do it and am not able to explain process or key points	I can sort of do it and am able to show process, but not able to identify/explain key math points	I can do it and able to both explain process and identify/explain math points	I'm great at doing it and am able to explain key math points accurately in a variety of problems

Unit 6 Definitions

<i>Term</i>	<i>Definition</i>	<i>Diagram</i>
Circle		
Radius		
Diameter		
Arc		
Semicircle		
Minor Arc		
Major Arc		
Chord		
Diameter		

Unit 6 Definitions

<i>Term</i>	<i>Definition</i>	<i>Diagram</i>
Tangent		
Point of Tangency		
Central Angle		
Inscribed Angle		
Circumference		

Unit 6 Conjectures

<i>Title</i>	<i>Conjecture</i>	<i>Diagram</i>
Chord Central Angles Conjecture	If two chords in a circle are congruent, then they determine two central angles that are...	
Chord Arcs Conjecture	If two chords in a circle are congruent, then their _____ are congruent.	
Perpendicular to a Chord Conjecture	The perpendicular segment from the center of a circle to a chord is the _____ of the chord.	
Chord Distance to Center Conjecture	Two congruent chords in a circle are _____ from the center of a circle.	
Perpendicular Bisector of a Chord Conjecture	The perpendicular bisector of a chord _____.	
Tangent Conjecture	A tangent to a circle _____ the radius drawn to the point of tangency.	
Tangent Segments Conjecture	Tangent segments to a circle from a point outside the circle are _____.	
Inscribed Angle Conjecture	The measure of an angle inscribed in a circle ____.	

Unit 6 Conjectures

<i>Title</i>	<i>Conjecture</i>	<i>Diagram</i>
Inscribed Angles Intercepting Arcs Conjecture	Inscribed angles that intercept the same arc _____.	
Angles Inscribed in a Semicircle Conjecture	Angles inscribed in a semicircle _____.	
Cyclic Quadrilateral Conjecture	The _____ angles of a cyclic quadrilateral are _____.	
Parallel Lines Intercepted Arcs Conjecture	Parallel lines intercept _____ arcs on a circle.	
Circumference Conjecture	If C is the circumference and d is the diameter of a circle, then there is a number π such that $C = ___$. If $d = 2r$ where r is the radius, then $C = ___$.	
Arc Length Conjecture	The length of an arc equals the _____ _____ _____.	

Additional Notes:

Notes

Notes

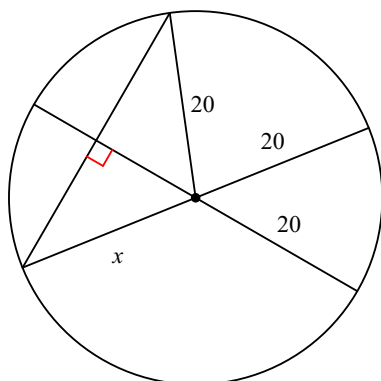
Chords and Tangents

Name: _____

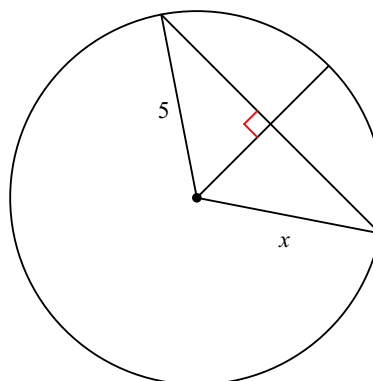
Date: _____ Period: _____

Find the length of the segment indicated. Round your answer to the nearest tenth if necessary.

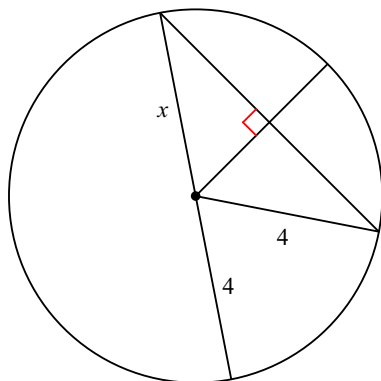
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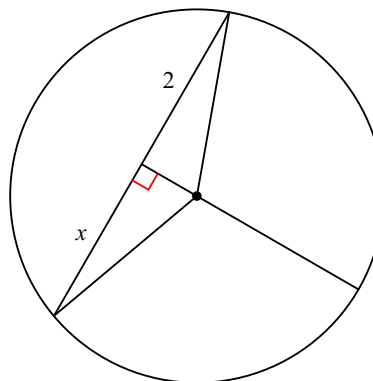
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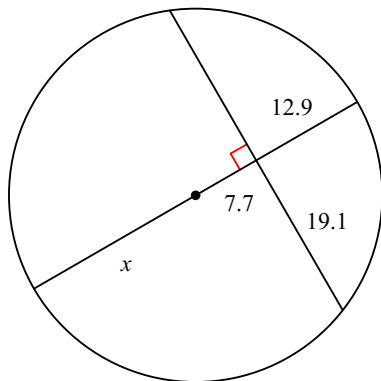
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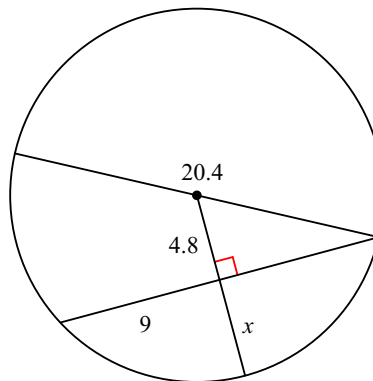
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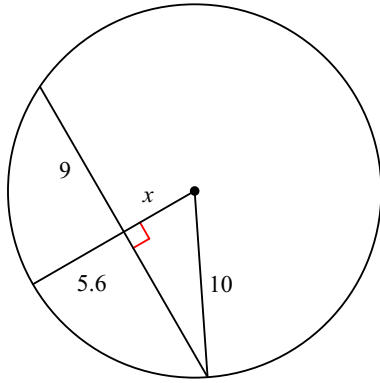
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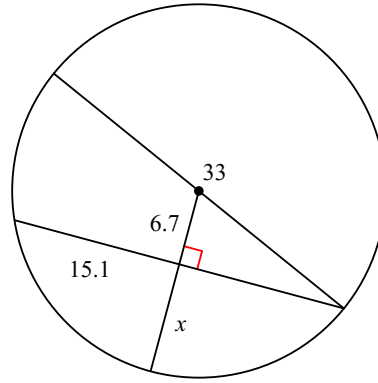
6)



7)

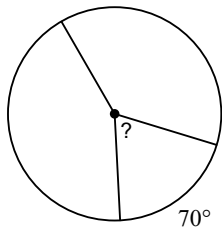


8)

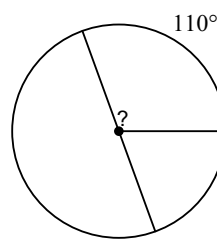


Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

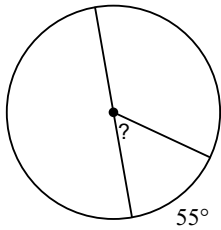
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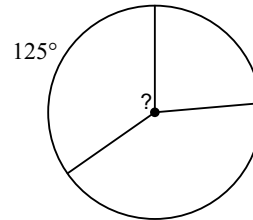
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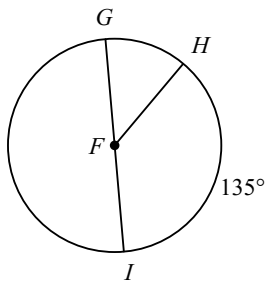
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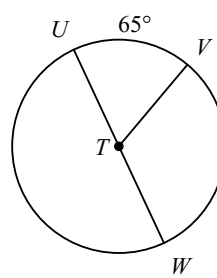
12)



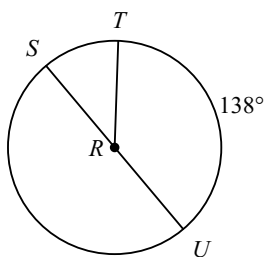
13) $m\angle GFH$



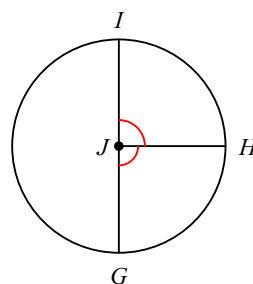
14) $m\angle VTW$



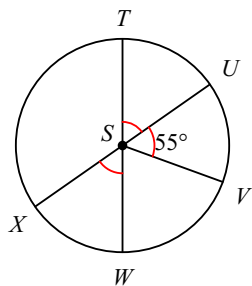
15) $m\angle SRT$



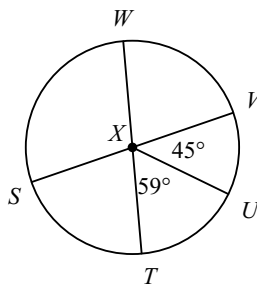
16) $m\angle HJG$



17) $m\angle XST$

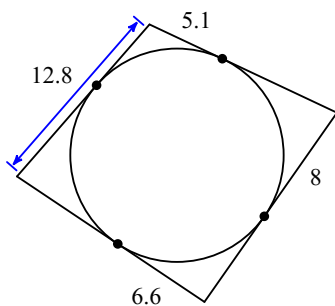


18) $m\angle WXV$

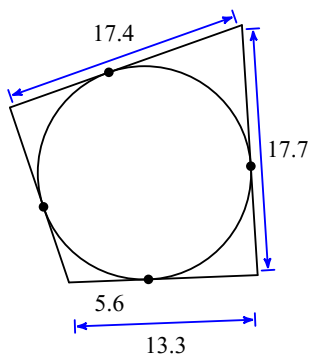


Find the perimeter of each polygon. Assume that lines which appear to be tangent are tangent.

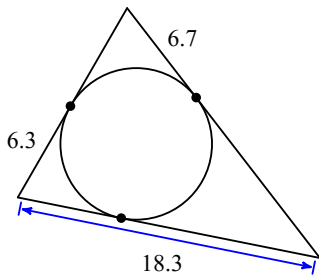
19)



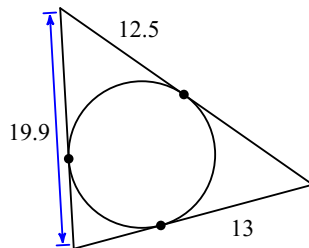
20)



21)

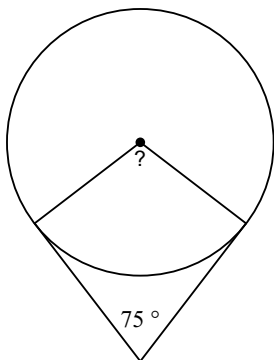


22)

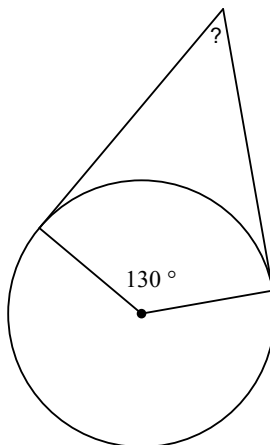


Find the angle measure indicated. Assume that lines which appear to be tangent are tangent.

23)

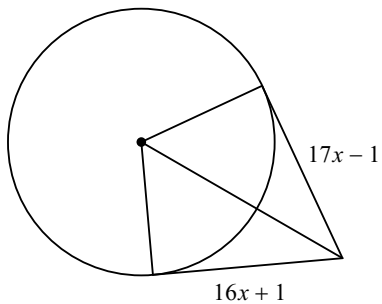


24)

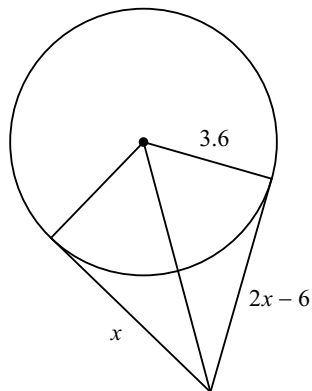


Solve for x . Assume that lines which appear to be tangent are tangent.

25)

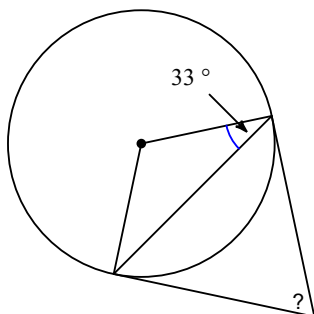


26)

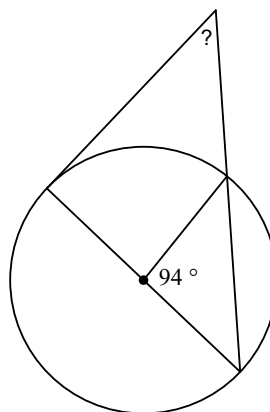


Find the angle measure indicated. Assume that lines which appear to be tangent are tangent.

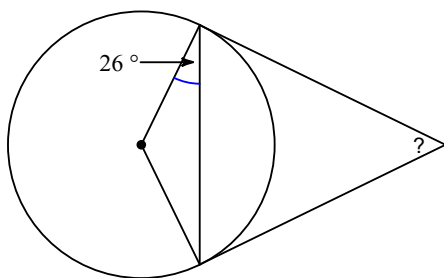
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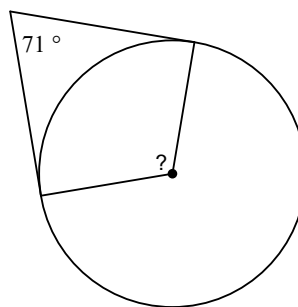
28)



29)



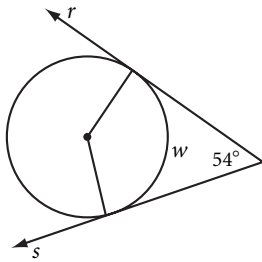
30)



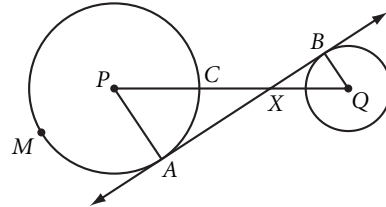
Lesson 6.1 • Tangent Properties

Name _____ Period _____ Date _____

1. Rays r and s are tangents. $w =$ _____

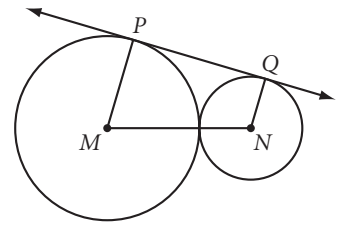


2. $\overleftrightarrow{AB}$ is tangent to both circles and $m\widehat{AMC} = 295^\circ$. $m\angle BQX =$ _____

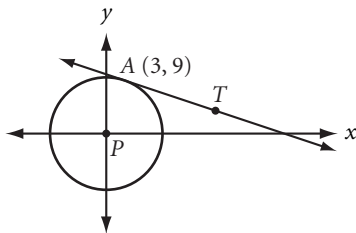


3. $\overleftrightarrow{PQ}$ is tangent to two externally tangent noncongruent circles, M and N .

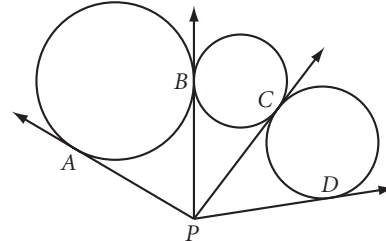
- $m\angle NQP =$ _____, $m\angle MPQ =$ _____
- What kind of quadrilateral is $MNQP$? Explain your reasoning.



4. $\overleftrightarrow{AT}$ is tangent to circle P . Find the equation of $\overleftrightarrow{AT}$.



5. $\overrightarrow{PA}$, $\overrightarrow{PB}$, $\overrightarrow{PC}$, and $\overrightarrow{PD}$ are tangents. Explain why $\overrightarrow{PA} \cong \overrightarrow{PD}$.



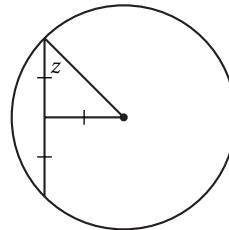
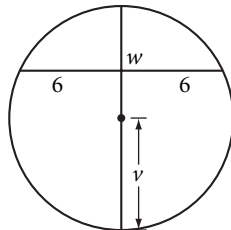
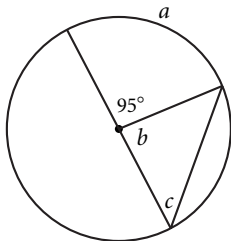
6. Circle A has diameter 16.4 cm. Circle B has diameter 6.7 cm.
- If A and B are internally tangent, what is the distance between their centers?
 - If A and B are externally tangent, what is the distance between their centers?
7. Construct a circle, P . Pick a point, A , on the circle. Construct a tangent through A . Pick a point, T , on the tangent. Construct a second tangent to the circle through T .

Lesson 6.2 • Chord Properties

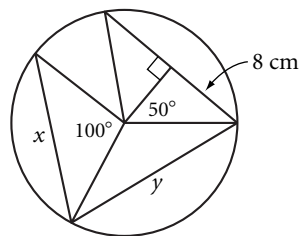
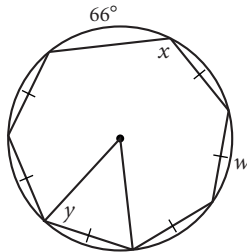
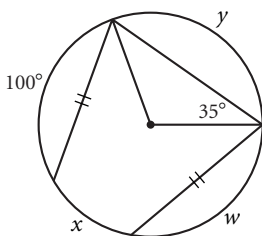
Name _____ Period _____ Date _____

In Exercises 1–6, find each unknown or write “cannot be determined.”

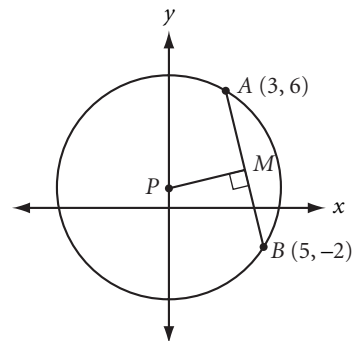
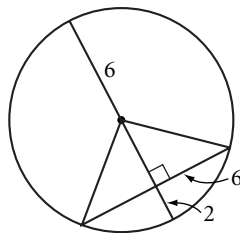
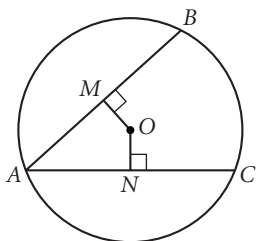
1. $a = \underline{\hspace{1cm}}, b = \underline{\hspace{1cm}},$
 $c = \underline{\hspace{1cm}}$
2. $w = \underline{\hspace{1cm}}, v = \underline{\hspace{1cm}}$
3. $z = \underline{\hspace{1cm}}$



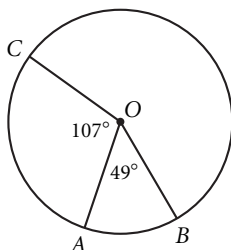
4. $w = \underline{\hspace{1cm}}, x = \underline{\hspace{1cm}},$
 $y = \underline{\hspace{1cm}}$
5. $w = \underline{\hspace{1cm}}, x = \underline{\hspace{1cm}},$
 $y = \underline{\hspace{1cm}}$
6. $x = \underline{\hspace{1cm}}, y = \underline{\hspace{1cm}}$



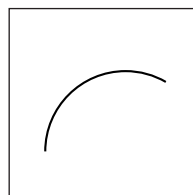
7. $\overline{AB} \cong \overline{AC}$. $\triangle AMON$ is a _____.
Justify your answer.
8. What's wrong with this picture?
9. Find the coordinates of P and M .
- γ



10. $m\widehat{AB} = \underline{\hspace{2cm}}$
 $m\widehat{ABC} = \underline{\hspace{2cm}}$
 $m\widehat{BAC} = \underline{\hspace{2cm}}$
 $m\widehat{ACB} = \underline{\hspace{2cm}}$



- 11.** Trace part of a circle onto patty paper. Fold to find the center. Explain your method.



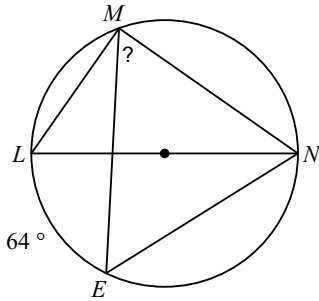
Practice: Inscribed Angles

Name: _____

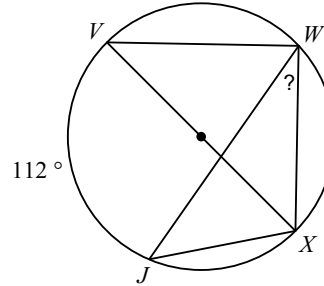
Date: _____ Period: _____

Find the measure of the arc or angle indicated.

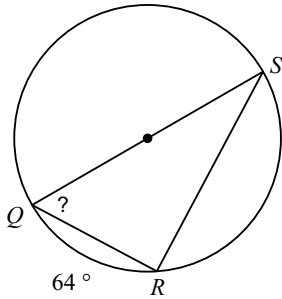
1)



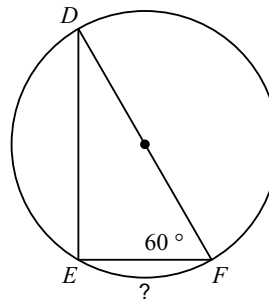
2)



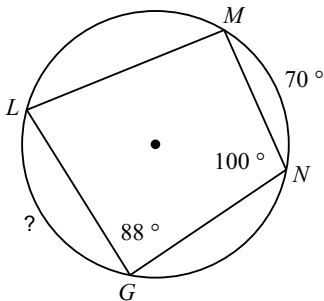
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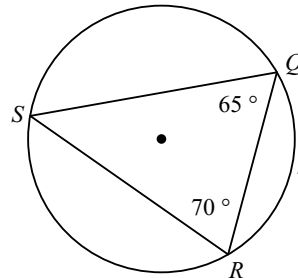
4)



5)

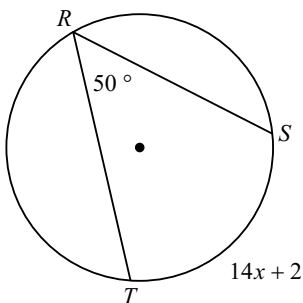


6)

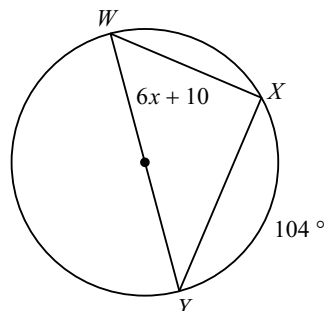


Solve for x .

7)



8)



Lesson 6.3 • Arcs and Angles

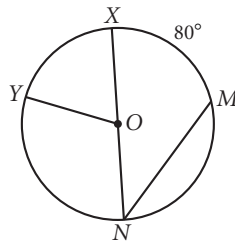
Name _____ Period _____ Date _____

1. $m\widehat{XM} = 80^\circ$

$m\angle XNM = \underline{\hspace{2cm}}$

$m\widehat{XN} = \underline{\hspace{2cm}}$

$m\widehat{MN} = \underline{\hspace{2cm}}$

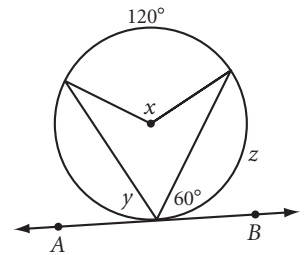


2. $\overleftrightarrow{AB}$ is a tangent.

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

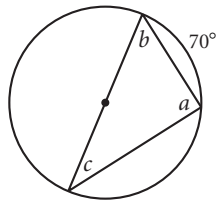
$z = \underline{\hspace{2cm}}$



3. $a = \underline{\hspace{2cm}}$

$b = \underline{\hspace{2cm}}$

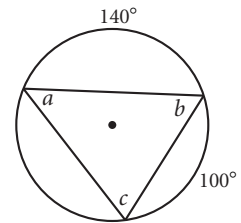
$c = \underline{\hspace{2cm}}$



4. $a = \underline{\hspace{2cm}}$

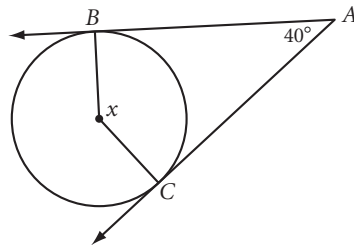
$b = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$



5. $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are tangents.

$x = \underline{\hspace{2cm}}$



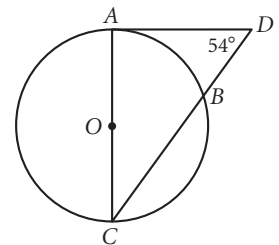
6. $\overleftrightarrow{AD}$ is a tangent. $\overleftrightarrow{AC}$ is a diameter.

$m\angle A = \underline{\hspace{2cm}}$

$m\widehat{AB} = \underline{\hspace{2cm}}$

$m\angle C = \underline{\hspace{2cm}}$

$m\widehat{CB} = \underline{\hspace{2cm}}$

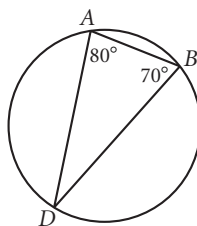


7. $m\widehat{AD} = \underline{\hspace{2cm}}$

$m\angle D = \underline{\hspace{2cm}}$

$m\widehat{AB} = \underline{\hspace{2cm}}$

$m\widehat{DAB} = \underline{\hspace{2cm}}$

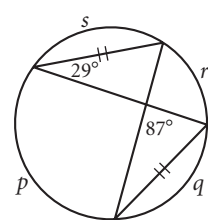


8. $p = \underline{\hspace{2cm}}$

$q = \underline{\hspace{2cm}}$

$r = \underline{\hspace{2cm}}$

$s = \underline{\hspace{2cm}}$



9. Find the lettered angle and arc measures. $\overleftrightarrow{AT}$ and $\overleftrightarrow{AZ}$ are tangents.

$a = \underline{\hspace{2cm}}$

$d = \underline{\hspace{2cm}}$

$g = \underline{\hspace{2cm}}$

$k = \underline{\hspace{2cm}}$

$b = \underline{\hspace{2cm}}$

$e = \underline{\hspace{2cm}}$

$h = \underline{\hspace{2cm}}$

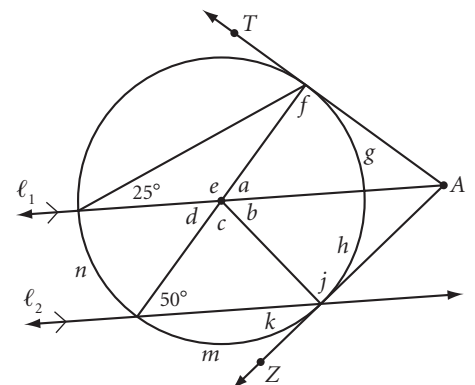
$m = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$

$f = \underline{\hspace{2cm}}$

$j = \underline{\hspace{2cm}}$

$n = \underline{\hspace{2cm}}$



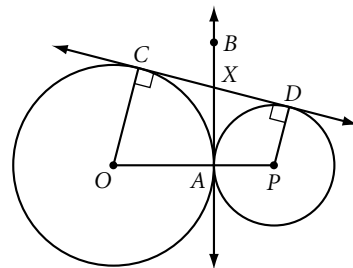
Lesson 6.4 • Proving Circle Conjectures

Name _____ Period _____ Date _____

In Exercises 1–4, complete each proof with a paragraph or a flowchart.

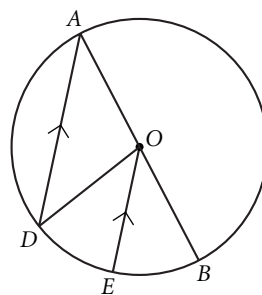
- 1. Given:** Circles O and P are externally tangent, with common tangents $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$

Show: $\overleftrightarrow{AB}$ bisects $\overleftrightarrow{CD}$ at X



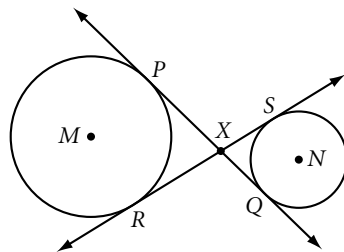
- 2. Given:** Circle O with diameter $\overline{AB}$ and chord $\overline{AD}$. $\overline{OE} \parallel \overline{AD}$.

Show: $\widehat{DE} \cong \widehat{BE}$



- 3. Given:** $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are tangent to both circles.

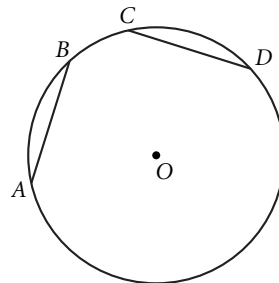
Show: $\overline{PQ} \cong \overline{RS}$.



- 4. Prove the converse of the Chord Arcs Conjecture:** If two arcs in a circle are congruent, then their chords are congruent. *Hint:* Draw radii.

Given: $\widehat{AB} \cong \widehat{CD}$

Show: $\overline{AB} \cong \overline{CD}$



Circumference Ratio**Find the diameter of each circle.**

1) circumference = 4π mi

2) circumference = 18π cm

3) circumference = 16π m

4) circumference = 14π yd

Find the radius of each circle.

5) circumference = 4π mi

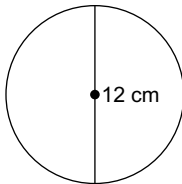
6) circumference = 8π cm

7) circumference = 12π mi

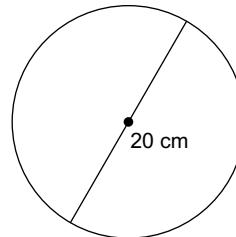
8) circumference = 10π m

Find the circumference of each circle.

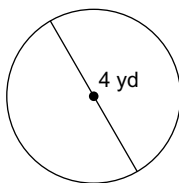
9)



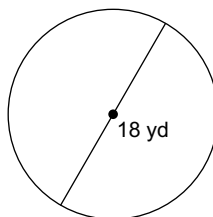
10)



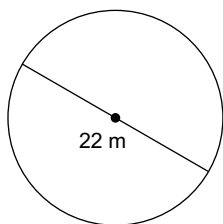
11)



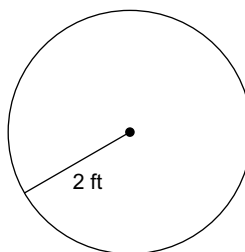
12)



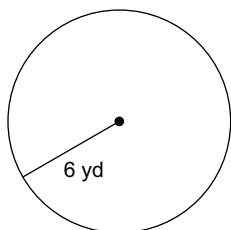
13)



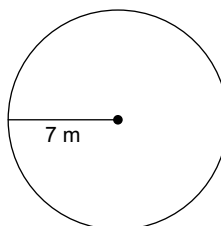
14)



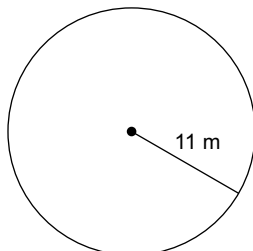
15)



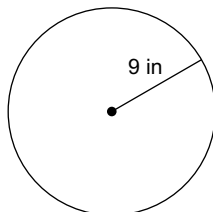
16)



17)



18)



Lesson 6.5 • The Circumference/Diameter Ratio

Name _____ Period _____ Date _____

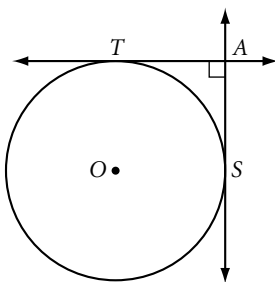
In Exercises 1–4, leave your answers in terms of π .

1. If $r = 10.5$ cm, find C .
2. If $C = 25\pi$ cm, find r .
3. What is the circumference of a circle whose radius is 30 cm?
4. What is the diameter of a circle whose circumference is 24π cm?

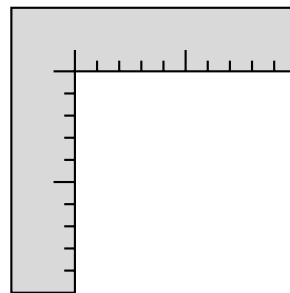
In Exercises 5–9, round your answer to the nearest 0.1 unit. Use the symbol $\approx$ to show that your answer is an approximation.

5. If $d = 9.6$ cm, find C .
6. If $C = 132$ cm, find d and r .
7. A dinner plate fits snugly in a square box with perimeter 48 inches. What is the circumference of the plate?
8. Four saucers are part of the same set as the dinner plate in Exercise 7. Each has a circumference of 15.7 inches. Will they fit, side by side, in the same square box? If so, how many inches will there be between the saucers for padding?

9. $\overleftrightarrow{AT}$ and $\overleftrightarrow{AS}$ are tangents. $AT = 12$ cm. What is the circumference of circle O ?



10. How can you use a large carpenter's square to find the circumference of a tree?



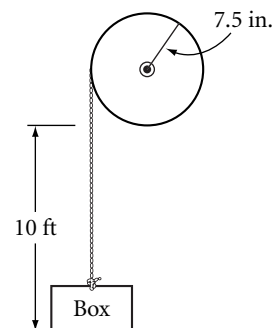
11. In order to increase the circumference of a circle from 16π cm to 20π cm, by how much must the diameter increase?

Lesson 6.6 • Around the World

Name _____ Period _____ Date _____

1. Alfonzo's Pizzeria bakes olive pieces in the outer crust of its 20-inch (diameter) pizza. There is at least one olive piece per inch of crust. How many olive pieces will you get in one slice of pizza? Assume the pizza is cut into eight slices.

2. To use the machine at right, you turn the crank, which turns the pulley wheel, which winds the rope and lifts the box. Through how many rotations must you turn the crank to lift the box 10 feet?

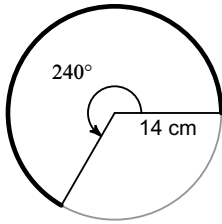


3. A satellite in *geostationary* orbit stays over the same spot on Earth. The satellite completes one orbit in the same time that Earth rotates once about its axis (23.93 hours). If the satellite's orbit has radius 4.23×10^7 m, calculate the satellite's orbital speed (tangential velocity) in meters per second.
4. You want to decorate the side of a cylindrical can by coloring a rectangular piece of paper and wrapping it around the can. The paper is 19 cm by 29 cm. Find the two possible diameters of the can to the nearest 0.01 cm. Assume the paper fits exactly.
5. As you sit in your chair, you are whirling through space with Earth as it moves around the sun. If the average distance from Earth to the sun is 1.4957×10^{11} m and Earth completes one revolution every 364.25 days, what is your "sitting" speed in space relative to the sun? Give your answer in km/h, rounded to the nearest 100 km/h.

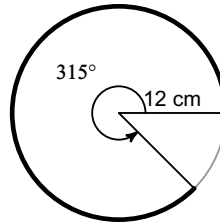
Arc Length

Find the length of each arc. Leave your answers in terms of π .

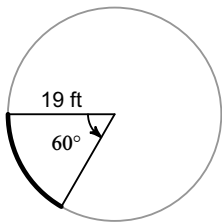
1)



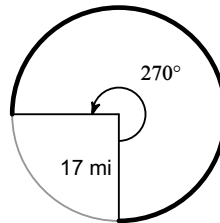
2)



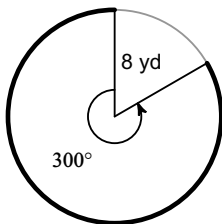
3)



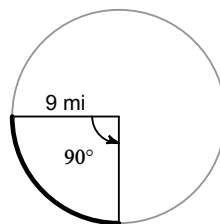
4)



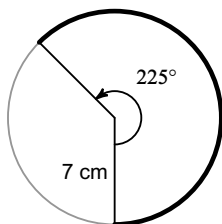
5)



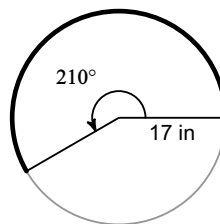
6)



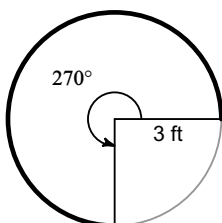
7)



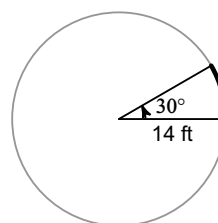
8)



9)



10)

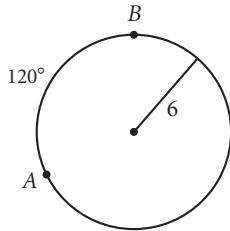


Lesson 6.7 • Arc Length

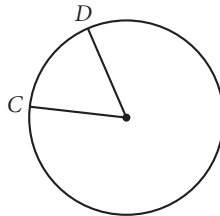
Name _____ Period _____ Date _____

In Exercises 1–10, leave your answers in terms of π .

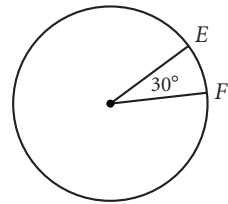
1. Length of $\widehat{AB}$ = _____



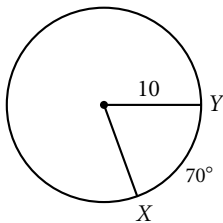
2. The circumference is 24π and $m\widehat{CD} = 60^\circ$. Length of $\widehat{CD}$ = _____



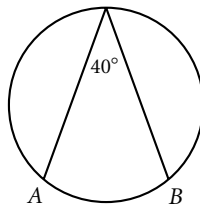
3. The length of $\widehat{EF}$ is 5π . Radius = _____



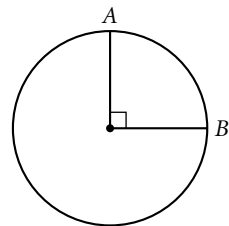
4. Length of $\widehat{XY}$ = _____



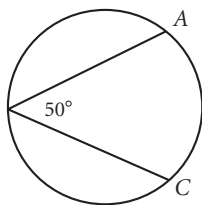
5. The radius is 20. Length of $\widehat{AB}$ = _____



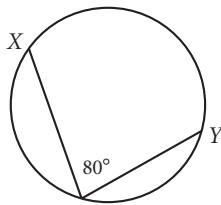
6. The circumference is 25π . Length of $\widehat{AB}$ = _____



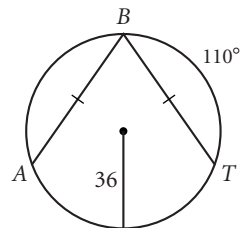
7. The diameter is 40. Length of $\widehat{AC}$ = _____



8. The length of $\widehat{XY}$ is 14π . Diameter = _____



9. Length of $\widehat{AB}$ = _____



10. A circle has an arc with measure 80° and length 88π . What is the diameter of the circle?

Unit 6 • Challenge Problems

1. (Target 6a)

Use a compass and straightedge to construct a circle and then to circumscribe a rhombus about the circle. Leave all construction marks. Explain the method you used and prove that the quadrilateral you constructed is a rhombus.

2. (Target 6a, 6b & 6c)

Find each of the following measures if possible, and explain how you found it. If a measure cannot be determined, write "cannot be determined."

$\overrightarrow{PS}$ and $\overrightarrow{PT}$ are tangent to circle O. $\overrightarrow{PS} \parallel \overrightarrow{QR}$.

$m\widehat{SR} = 40^\circ$, $m\widehat{MLO} = 38^\circ$, $m\widehat{TOI} = 48^\circ$, $m\widehat{AB} = 110^\circ$

$m\angle P = \underline{\hspace{1cm}}$

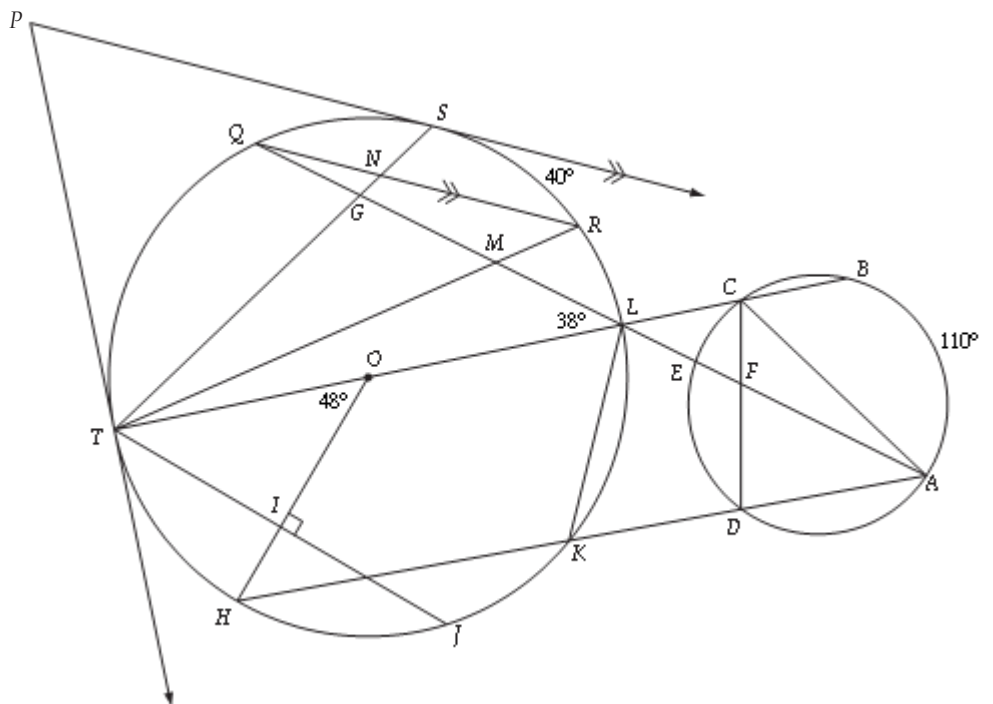
$m\angle DAF = \underline{\hspace{1cm}}$

$m\angle RML = \underline{\hspace{1cm}}$

$m\widehat{KL} = \underline{\hspace{1cm}}$

$m\widehat{HJ} = \underline{\hspace{1cm}}$

$m\widehat{CE} = \underline{\hspace{1cm}}$



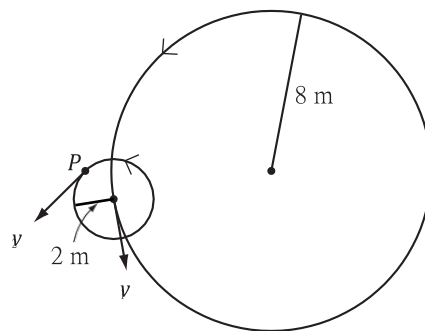
Unit 6 • Challenge Problems

3. (Target 6a, 6e & 6f)

Tangential velocity is a measure of the distance an object travels along a circular path in a given amount of time. For example, if an object covers a circular path with circumference 20π meters in 5 seconds, then its tangential velocity is 4π meters per second. A carnival ride at Hold On Fun Park has seats attached to the circumference of a horizontal wheel.

This wheel is in turn attached at its center to the circumference of a larger horizontal wheel. The larger wheel has a radius of 8 m and rotates at a speed of 5 rpm (rotations per minute). The smaller wheel has a radius of 2 m and rotates at 20 rpm.

You, in the seat, feel the combined effect of both rotations. For example, suppose you are sitting at point P in the diagram. If $\vec{v}_1$ is the tangential velocity of the point where the larger circle attaches to the center of the smaller circle, and $\vec{v}_2$ is the tangential velocity of point P , the velocity you experience will be the sum of $\vec{v}_1$ and $\vec{v}_2$. Find the maximum and minimum velocities you will experience while on the ride. (Give the magnitude only, not the direction.) Explain your reasoning.



4. (Target 6e)

If you added 18 feet to a rope that is taught around the world (24,000 miles), and you stretched it out evenly around the world, how high off the ground would the rope be?

Unit 6 • Challenge Problems

5. (*Target 6f*)

In the lobby of the Natural World Science Center, a long pendulum is suspended from the ceiling. The pendulum cable is 20 meters long, and the heavy bob swings through an arc of 15° . The time t , in seconds, for one complete swing (back and forth) is given by the formula $t = 2\pi\sqrt{\frac{l}{9.8}}$, where l is the cable length in meters.

a. What is the average speed of the pendulum bob in meters per second? Explain the method you used to find the answer.

b. How many kilometers does the bob travel in one year? Explain the method you used to find the answer.

6. (*Targets 6f*)

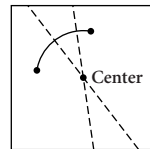
The longitude of a geographic location is measured in degrees east (E) or west (W) from the prime meridian (0°) to the International Date Line (180°). Quito, Ecuador, on the west coast of South America, has a longitude of 78.5° W. Singapore, on the Malay Peninsula in Asia, has a longitude of 104° E. Both cities are very close to the equator. (Assume they are on the equator for this problem.) The diameter of Earth at the equator is 7928 miles. What is the shortest distance from Quito to Singapore? Sketch a circle to represent the equator and mark the appropriate information. Explain how you found your answer.

3. $z = 45^\circ$
4. $w = 100^\circ$, $x = 50^\circ$, $y = 110^\circ$
5. $w = 49^\circ$, $x = 122.5^\circ$, $y = 65.5^\circ$
6. $x = 16$ cm, y cannot be determined
7. Kite. Possible explanation: $\overline{OM} \cong \overline{ON}$ because congruent chords $\overline{AB}$ and $\overline{AC}$ are the same distance from the center. $\overline{AM} \cong \overline{AN}$ because they are halves of congruent chords. So, $AMON$ has two pairs of adjacent congruent sides and is a kite.
8. The perpendicular segment from the center of the circle bisects the chord, so the chord has length 12 units. But the diameter of the circle is 12 units, and the chord cannot be as long as the diameter because it doesn't pass through the center of the circle.

9. $P(0,1)$, $M(4, 2)$

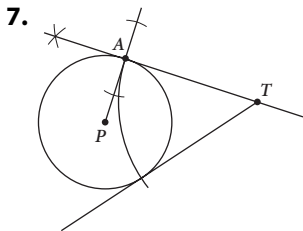
10. $m\widehat{AB} = 49^\circ$, $m\widehat{ABC} = 253^\circ$, $m\widehat{BAC} = 156^\circ$,
 $m\widehat{ACB} = 311^\circ$

11. Possible answer: Fold and crease to match the endpoints of the arc. The crease is the perpendicular bisector of the chord connecting the endpoints. Fold and crease so that one endpoint falls on any other point on the arc. The crease is the perpendicular bisector of the chord between the two matching points. The center is the intersection of the two creases.



LESSON 6.1 • Tangent Properties

1. $w = 126^\circ$
2. $m\angle BQX = 65^\circ$
3. a. $m\angle NQP = 90^\circ$, $m\angle MPQ = 90^\circ$
 b. Trapezoid. Possible explanation: $\overline{MP}$ and $\overline{NQ}$ are both perpendicular to $\overline{PQ}$, so they are parallel to each other. The distance from M to $\overline{PQ}$ is MP , and the distance from N to $\overline{PQ}$ is NQ . But the two circles are not congruent, so $MP \neq NQ$. Therefore, $\overline{MN}$ is not a constant distance from $\overline{PQ}$ and they are not parallel. Exactly one pair of sides is parallel, so $MNQP$ is a trapezoid.
4. $y = -\frac{1}{3}x + 10$
5. Possible answer: Tangent segments from a point to a circle are congruent. So, $\overline{PA} \cong \overline{PB}$, $\overline{PB} \cong \overline{PC}$, and $\overline{PC} \cong \overline{PD}$. Therefore, $\overline{PA} \cong \overline{PD}$.
6. a. 4.85 cm
 b. 11.55 cm



LESSON 6.2 • Chord Properties

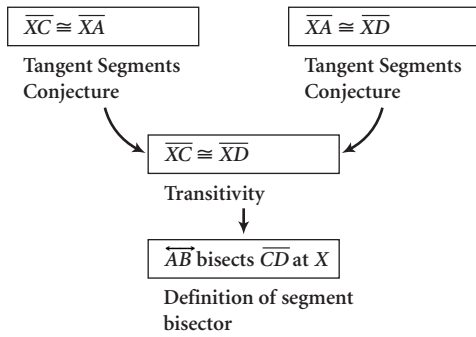
1. $a = 95^\circ$, $b = 85^\circ$, $c = 47.5^\circ$
2. v cannot be determined, $w = 90^\circ$

LESSON 6.3 • Arcs and Angles

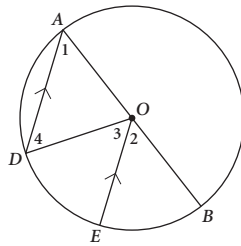
1. $m\angle XNM = 40^\circ$, $m\widehat{XN} = 180^\circ$, $m\widehat{MN} = 100^\circ$
2. $x = 120^\circ$, $y = 60^\circ$, $z = 120^\circ$
3. $a = 90^\circ$, $b = 55^\circ$, $c = 35^\circ$
4. $a = 50^\circ$, $b = 60^\circ$, $c = 70^\circ$
5. $x = 140^\circ$
6. $m\angle A = 90^\circ$, $m\widehat{AB} = 72^\circ$, $m\angle C = 36^\circ$, $m\widehat{CB} = 108^\circ$
7. $m\widehat{AD} = 140^\circ$, $m\angle D = 30^\circ$, $m\widehat{AB} = 60^\circ$,
 $m\widehat{DAB} = 200^\circ$
8. $p = 128^\circ$, $q = 87^\circ$, $r = 58^\circ$, $s = 87^\circ$
9. $a = 50^\circ$, $b = 50^\circ$, $c = 80^\circ$, $d = 50^\circ$, $e = 130^\circ$,
 $f = 90^\circ$, $g = 50^\circ$, $h = 50^\circ$, $j = 90^\circ$, $k = 40^\circ$,
 $m = 80^\circ$, $n = 50^\circ$

LESSON 6.4 • Proving Circle Conjectures

1. Flowchart Proof



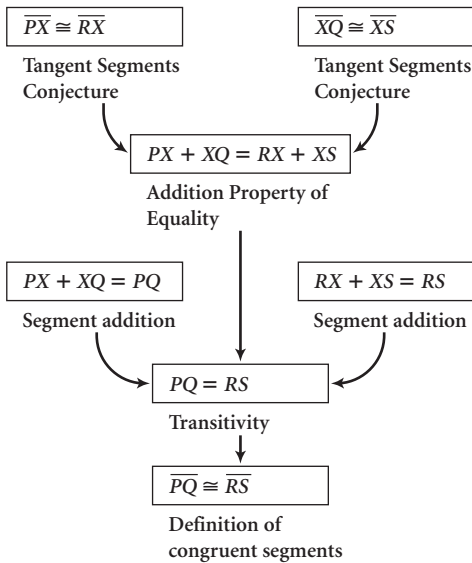
2. Angles are numbered for reference.



Paragraph Proof

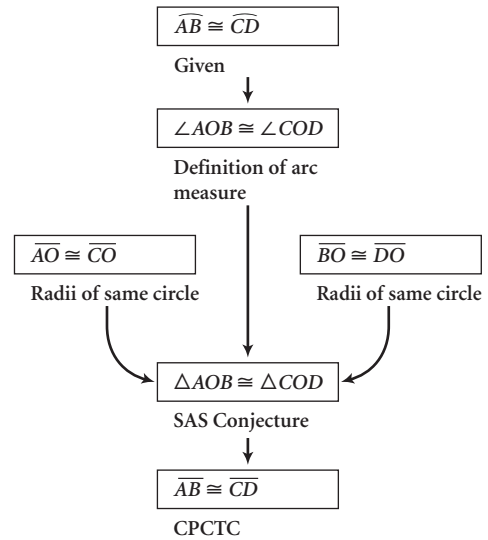
It is given that $\overline{OE} \parallel \overline{AD}$, so $\angle 2 \cong \angle 1$ by the CA Conjecture. Because $\overline{OA}$ and $\overline{OD}$ are radii, they are congruent, so $\triangle AOD$ is isosceles. Therefore $\angle 4 \cong \angle 1$ by the IT Conjecture. Both $\angle 2$ and $\angle 4$ are congruent to $\angle 1$, so $\angle 2 \cong \angle 4$. By the AIA Conjecture, $\angle 4 \cong \angle 3$, so $\angle 2 \cong \angle 3$. The measure of an arc equals the measure of its central angle, so because their central angles are congruent, $\overline{DE} \cong \overline{BE}$.

3. Flowchart Proof



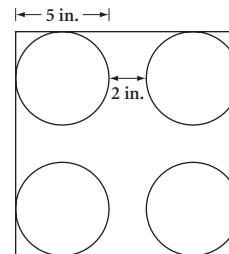
4. Flowchart Proof

Construct radii $\overline{AO}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$.

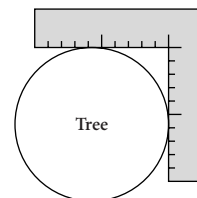


LESSON 6.5 • The Circumference/Diameter Ratio

- $C = 21\pi$ cm
- $r = 12.5$ cm
- $C = 60\pi$ cm
- $d = 24$ cm
- $C \approx 30.2$ cm
- $d \approx 42.0$ cm, $r \approx 21.0$ cm
- $C \approx 37.7$ in.
- Yes; about 2.0 in.



- $C \approx 75.4$ cm
- Press the square against the tree as shown. Measure the tangent segment on the square. The tangent segment is the same length as the radius. Use $C = 2\pi r$ to find the circumference.



- 4 cm

LESSON 6.6 • Around the World

1. At least 7 olive pieces
2. About 2.5 rotations
3. $\frac{(2\pi \cdot 4.23 \cdot 10^7)}{(60 \cdot 60 \cdot 23.93)} \approx 3085$ m/s (about 3 km/s or just under 2 mi/s)
4. 6.05 cm or 9.23 cm
5. Sitting speed = $\frac{(2\pi \cdot 1.4957 \cdot 10^{11}/10^3)}{(364.25 \cdot 24)} \approx 107,500$ km/h

LESSON 6.7 • Arc Length

- | | |
|-----------------------|-----------------------------------|
| 1. 4π | 2. 4π |
| 3. 30 | 4. $\frac{35\pi}{9}$ |
| 5. $\frac{80\pi}{9}$ | 6. 6.25π or $\frac{25\pi}{4}$ |
| 7. $\frac{100\pi}{9}$ | 8. 31.5 |
| 9. 22π | 10. 396 |

EXPLORATION • Intersection Secants, Tangents, and Chords

1. $x = 21^\circ$
2. $m\widehat{DC} = 70^\circ$, $m\widehat{ED} = 150^\circ$
3. $m\widehat{DC} = 114^\circ$, $m\angle DEC = 66^\circ$
4. $m\angle BCE = 75^\circ$, $m\widehat{BAC} = 210^\circ$
5. $x = 80^\circ$, $y = 110^\circ$, $z = 141^\circ$
6. $x = 34^\circ$, $y = 150^\circ$, $z = 122^\circ$
7. $x = 112^\circ$, $y = 68^\circ$, $z = 53^\circ$
8. $x = 28^\circ$, $y = 34.5^\circ$

Answers to Chords and Tangents

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| 1) 20 | 2) 5 | 3) 4 | 4) 2 |
| 5) 20.6 | 6) 5.4 | 7) 4.4 | 8) 9.8 |
| 9) 70° | 10) 110° | 11) 55° | 12) 125° |
| 13) 45° | 14) 115° | 15) 42° | 16) 90° |
| 17) 125° | 18) 76° | 19) 54.8 | 20) 61.4 |
| 21) 50 | 22) 65.8 | 23) 105° | 24) 50° |
| 25) 2 | 26) 6 | 27) 66° | 28) 47° |
| 29) 52° | 30) 109° | | |

Answers to Practice: Inscribed Angles

- | | | | |
|---------------|---------------|---------------|---------------|
| 1) 58° | 2) 34° | 3) 58° | 4) 60° |
| 5) 94° | 6) 90° | 7) 7 | 8) 7 |

Answers to Circumference Ratio

- | | | | |
|---------------|----------------|----------------|----------------|
| 1) 4 mi | 2) 18 cm | 3) 16 m | 4) 14 yd |
| 5) 2 mi | 6) 4 cm | 7) 6 mi | 8) 5 m |
| 9) 12π cm | 10) 20π cm | 11) 4π yd | 12) 18π yd |
| 13) 22π m | 14) 4π ft | 15) 12π yd | 16) 14π m |
| 17) 22π m | 18) 18π in | | |

Answers to Arc Length

- | | | | |
|-------------------------|-------------------------|-------------------------|--------------------------|
| 1) $\frac{56\pi}{3}$ cm | 2) 21π cm | 3) $\frac{19\pi}{3}$ ft | 4) $\frac{51\pi}{2}$ mi |
| 5) $\frac{40\pi}{3}$ yd | 6) $\frac{9\pi}{2}$ mi | 7) $\frac{35\pi}{4}$ cm | 8) $\frac{119\pi}{6}$ in |
| 9) $\frac{9\pi}{2}$ ft | 10) $\frac{7\pi}{3}$ ft | | |