Unit 8:

Exponential Equations

Name: ________________________________

Goals:

Target A: I can write, represent, evaluate and solve exponential functions using a table, graph, or situation.

Target B: I can explain the properties of a negative, fractional and zero exponents.

Target C: I can identify which situations can be modeled with an exponential functions.

Resources:

http://mrnohner.com/exposeq.html

Unit Test Date	

Homework	
Page(s)	Problems

Homework Grading	
Homework completed	Grade
100% AND 50% of challenge problems Or unit project	4
75%	3
50%	2
49% or less	1

Most Valuable Retakes	
Retake	Done?
1.	
2.	
3.	

<table>
<tr>
<td>
I can find the percentage gain or loss given an exponential equation.

1) Given this form $y = a \cdot b^x$ ignore the “a” value, it is only the starting point and focus only on the “b” value.
2) Take the “b” value minus 1.00.
    a) Note: at this point it will be positive if it is exponential growth and negative if it is exponential decay
3) Take that value and multiply by 100 to find what that is in percentage form.
</td>
<td>
Example 1: What is the percentage lost or gained in the following exponential equations?

1) $y = 5(1.037)^x$

2) $y = 199(.92)^x$

3) $y = 5.43(2.43)^x$
</td>
</tr>
<tr>
<td>
I can find the exponential equation given a situation:

1) $y = a \cdot b^x$ is the form for exponential functions.
2) Identify the starting value. That goes in for “a”.
3) Is it exponential growth or decay?
    a) If it is growth divide the percentage growth by 100 and add it to 1.00 to find the multiplier which is the b value.
    b) If it is decay divide the percent decay by 100 and subtract it from 1.00 to find the multiplier which is the b value.
</td>
<td>
Example 1: You are running a bakery and each year the cost of flour goes up by 3 percent. Currently it costs 2 dollars a pound. What is the equation that models this situation?

Example 2: You are running an experiment with 10 gallons of water poured into a small pool. 7 percent of it evaporates every hour. What is the equation that models this situation?
</td>
</tr>
</table>

I can use exponential equations to model situations and provided estimates. 1) Determine if they are asking you to solve for a "y" value or an "x" value. a) If they are asking you to solve for a "y" value, plug the "x" value into your equation and evaluate. b) If they are asking you to solve an equation for an "x" value, then at this point in our mathematical career we have to make a table. Using your calculator follow these steps. Click "y =", type in your equation, click "2nd", click "graph". This will actually bring you to a table. Scroll around until your equation has an output that is closest to the output given.	Example Problem 1: 5,500 people started running the 5k this Saturday. Every minute though 1% of them dropped out of the race. 1) What is the equation that models this situation? 2) At 20 minutes, how many participants are still running? 3) When are there less than 5000 people still competing?
I know the exponential power rules:	

Today's Target:

Today’s Target:

Today's Target:

Today's Target:

Today's Target:

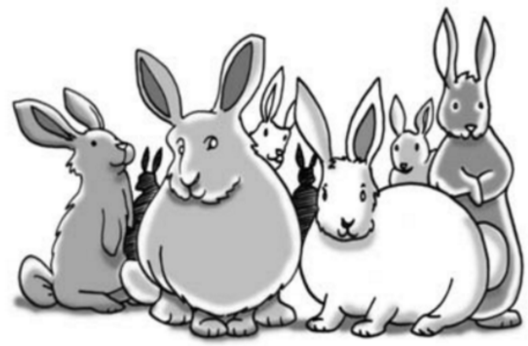
Today’s Target:

Today's Target:

Today's Target:

Introduction into Exponential Functions

You decide to start a rabbit farm. Starting out with only two rabbits you watch how their population grows. Every pair of rabbits has two babies every single month.



1) Create a diagram of the rabbit population (to 5 months)

2) Create a table that represents the number of rabbits (up to 12 months)

3) Use your table and diagram to find an equation that matches this data. **(HINT:** you can check your work by using your calculator to see if it produces the same table of information**)**

4) You want to compare the population growth of rabbits to other species. Below are some other animals and how many offspring they can produce every month.

For each animal:

Draw a diagram of the growth, **Make a table of the results** and **Find an equation**

a) What if you start with 2 crickets. Each pair of crickets has 10 babies a month.

b) What if you start with 6 moles. Each pair has 4 babies a month.

c) What if you start with 4 chickens. Each pair has 6 babies a month.

Exponential Decay

1. **Step 1:** Gather the data

Height of Drop	Height of Rebound	Rebound Ratio (Take the height of the rebound divided by the height of the drop)
150		
130		
110		
100		
80		
60		
40		
20		

2. **Step Two**: Find the rebound ratio. Take the mean of the rebound ratios above to get the overall rebound ratio.

   a. What was the rebound ratio for the ball your team used?

   b. Did the height you dropped the ball from affect this ratio?

   c. If you were to use the same ball again and drop it from *any* height, could you predict its rebound height? Explain.

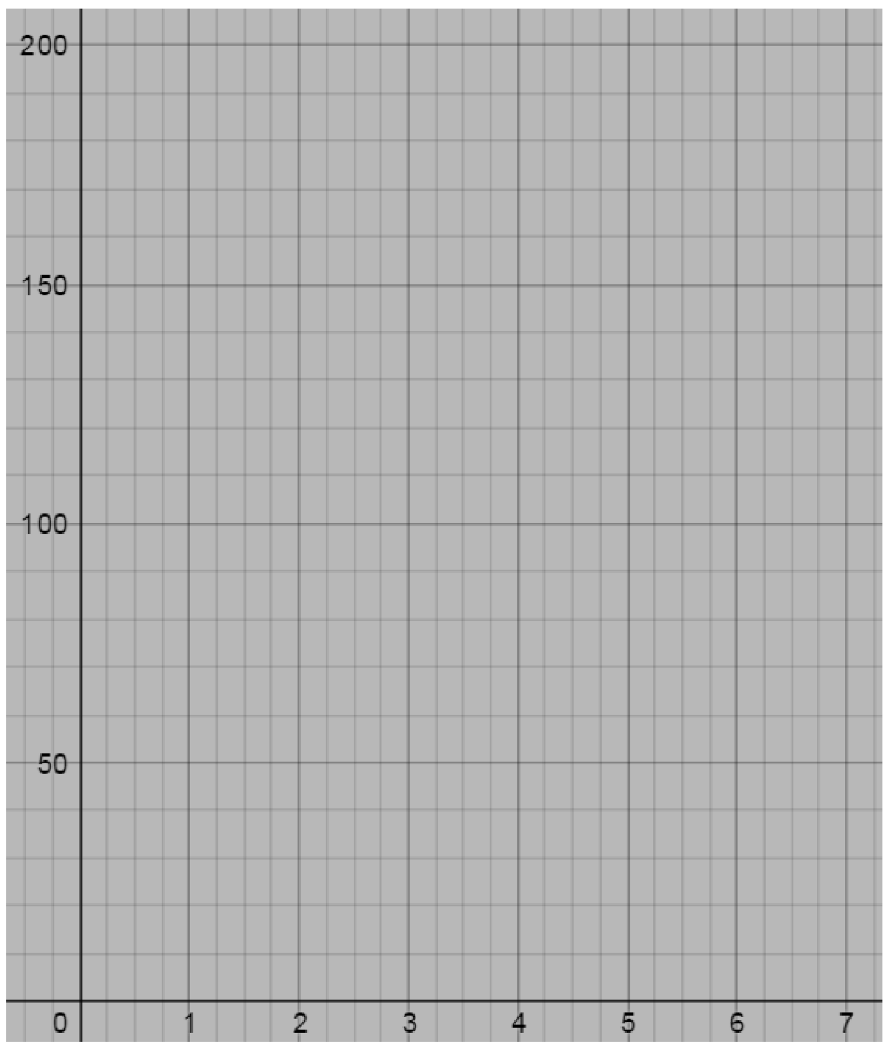
3. **Step 3:** Find the equation that models the data

a. To write ANY exponential equation you need just two values: the starting value and the multiplier that describes the growth. Take the mean rebound ratio from the table above and use that as your multiplier. What is it for your data?

b. If we wanted to model us dropping the ball from 200 cm, then what is the equation that models this situation?

4. **Step 4:** Create a table of values that models this the height after multiple bounces. Then graph the data.

Bounce Number	Rebound Height
0	
1	
2	
3	
4	
5	
6	



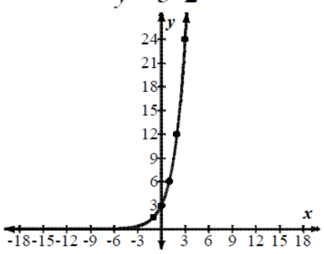
Example 1 Graph $y = 3 \cdot 2^x$

Make a table of values.

x	-1	0	1	2	3
y	1.5	3	6	12	24

Plot the points and connect them to form a smooth curve.

$y = 3 \cdot 2^x$



This is called an **increasing** exponential curve.

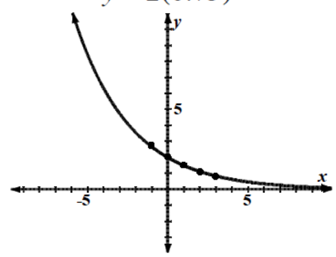
Example 2 Graph $y = 2(0.75)^x$

Make a table of values using a calculator.

x	-1	0	1	2	3
y	2.7	2	1.5	1.1	0.8

Plot the points and connect them to form a smooth curve.

$y = 2(0.75)^x$



This is called a **decreasing** exponential curve.

SOLVING EXPONENTIAL GROWTH AND DECAY PROBLEMS

Example 1

Movie tickets now average \$9.75 a ticket, but are increasing 15% per year. How much will they cost 5 years from now?

The equation to use is: $y = ab^x$. The initial value $a = 9.75$. The multiplier b is always found by adding the percent increase (as a decimal) to the number "one," so $b = 1 + 0.15 = 1.15$. The time is $x = 5$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 9.75(1.15)^5 \approx 19.61$. In five years movie tickets will average about \$19.61.

Example 2

A powerful computer is purchased for \$2000, but on the average loses 20% of its value each year. How much will it be worth 4 years from now?

The equation to use is: $y = ab^x$. The initial value $a = 2000$. In this case the value is decreasing so multiplier b is always found by subtracting the percent decrease from the number "one," so $b = 1 - 0.2 = 0.8$. The time is $x = 4$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 2000(0.8)^4 = 819.2$. In four years the computer will only be worth \$819.20.

Exponential Situations PART 1

1) The number of bacteria in a colony at noon is 180. This species of bacteria grows at a rate of 22% an hour. How many will be present at 8 p.m.?

2) A house purchased at $226,000 has lost 4% of its value each year for the past five years, what is it worth now?

3) A 1970 comic originally was sold for $0.35. Since then it has appreciated (gained) 10% a year. How much would it be worth if the owner sold it in 2010?

4) A Honda Accord typically depreciates at 14.5% a year. Six years ago it was purchased for $21,000, what is it worth now?

5) Inflation is at a rate of 3.5% per year. Today your favorite bread costs $3.79. What did it cost 10 years ago?

6) A two bedroom house in Nashville is worth $110,000 dollars. If it appreciates at 2.75%, how many years until it is worth over $200,000 dollars. I would create a table.

7) Ryan’s motorcycle is now worth $4,000. It has decreased in value by 12% for the last seven years he has owned it. How much was it worth originally?

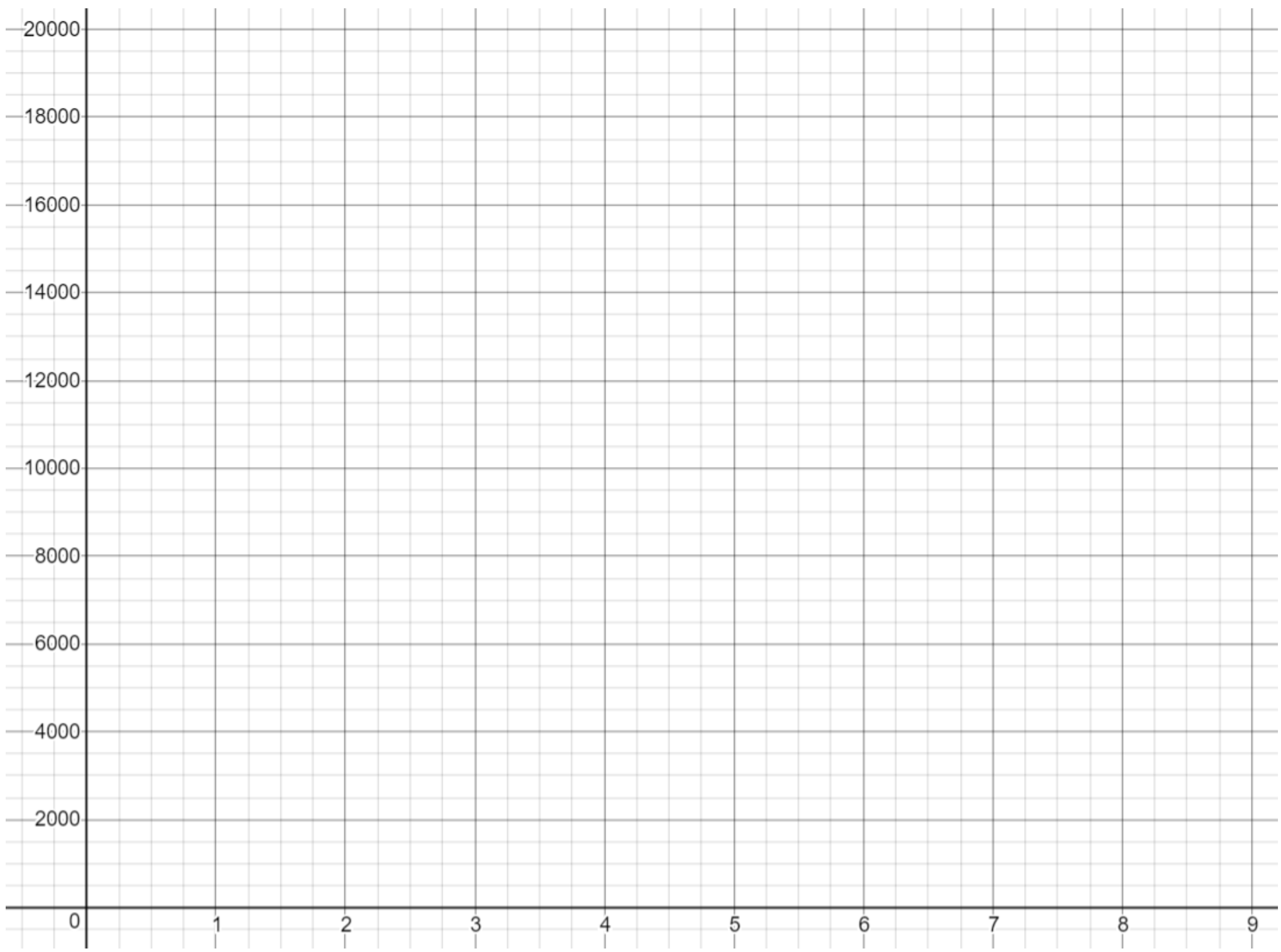
AN APPLICATION: CHOOSING A CAR

Most cars decrease in value after you leave the dealer. However, some cars are now considered "classics" and actually increase in value. You have the choice of owning two cars: A 2006 Mazda Maita which is worth $19,000 but is depreciating 10% per year, or a classic 1970 Ford Mustang which is worth $11,500 and is increasing in value by 6% each year. Your tasks:

a. Write an equation to represent the value of each car over time.

b. Create tables and draw a graph to represent the value of each car for ten years on the same set of axes.

c. Use your graph to determine approximately when the Mazda and the Ford have the same value.



<u>Exponential Situations Part 2</u>

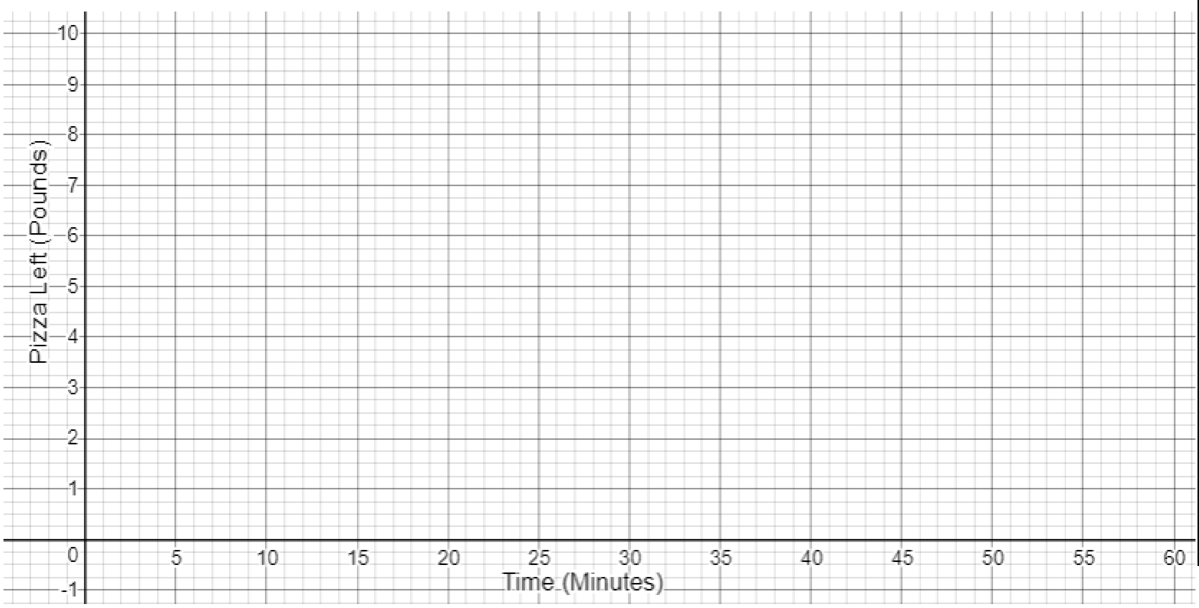
You are eating a full family sized Costco combination pizza and reading on your hammock. You fall asleep and the pizza falls to the ground below you. 3 ants find the pizza and begin gathering. The pizza weighs roughly 10 pounds to start.

1a) If you wake up 60 minutes later and there is only 1.6 pounds of pizza left. Can you figure out which of these four would be the multiplier that models this scenario?

i) 0.99 ii) 0.79 iii) .097 iv) .97

1b) Your answer for ***<u>1a</u>*** means that you have a loss of what percent every minute?

1c) Write the equation that models this situation, fill in the table and graph.



Time (minutes)	Pizza Remaining (Pounds)
5	
10	
15	
20	
25	
30	
35	
40	
45	
50	
55	
60	

2. You are looking into a loan and two companies offer different rates. Company A charges 2% every month for 3 years on your 5,000 dollar loan. Company B charges 24% every year for 3 years for your loan. (Hint: Be careful with what power you are raising the expression to)

a. Company A Total Loan Amount:

b. Company B Total Loan Amount:

c. Who Should You Choose?

3. The number of people on Earth grows at about 1.5% according to world surveys. Assuming that figure holds true, how long until the world population of 7,300,000,000 people reaches 8,000,000,000?

a. What is the equation that models this situation

b. Use a table to find the result.

1. Finding a date to prom is a difficult endeavour. Being the mathematician you are you decide to send out a survey to the entire student body trying to score a date. Every question you add to the survey eliminates 17% of the remaining potential prom dates. How many questions would you have to ask to bring the entire school of 1,724 students down to the 1 perfect match?

a. What is the equation that models this situation

b. The solution is:

2. Coffee is a fantastic drink. Studies show that the warmth and the caffeine stimulates the heart and tricks the brain into believing that your date is actually more amusing and attractive than they really are. When brewed it begins with a temperature of 200 degrees and after a minute is 195 degrees.

a. What is the multiplier and percentage of heat loss each minute for the coffee?

b. When does the coffee reach the prefered drinking temperature of 155 degrees?

c. How warm is the coffee after 120 minutes? Does this answer make sense in the context of this problem?

3. You are caller number 9 and the DJ says you get to pick between two prizes. Prize 1 pays out 250 dollars every single week. Prize two is 100 dollars that grows by 6% every week.

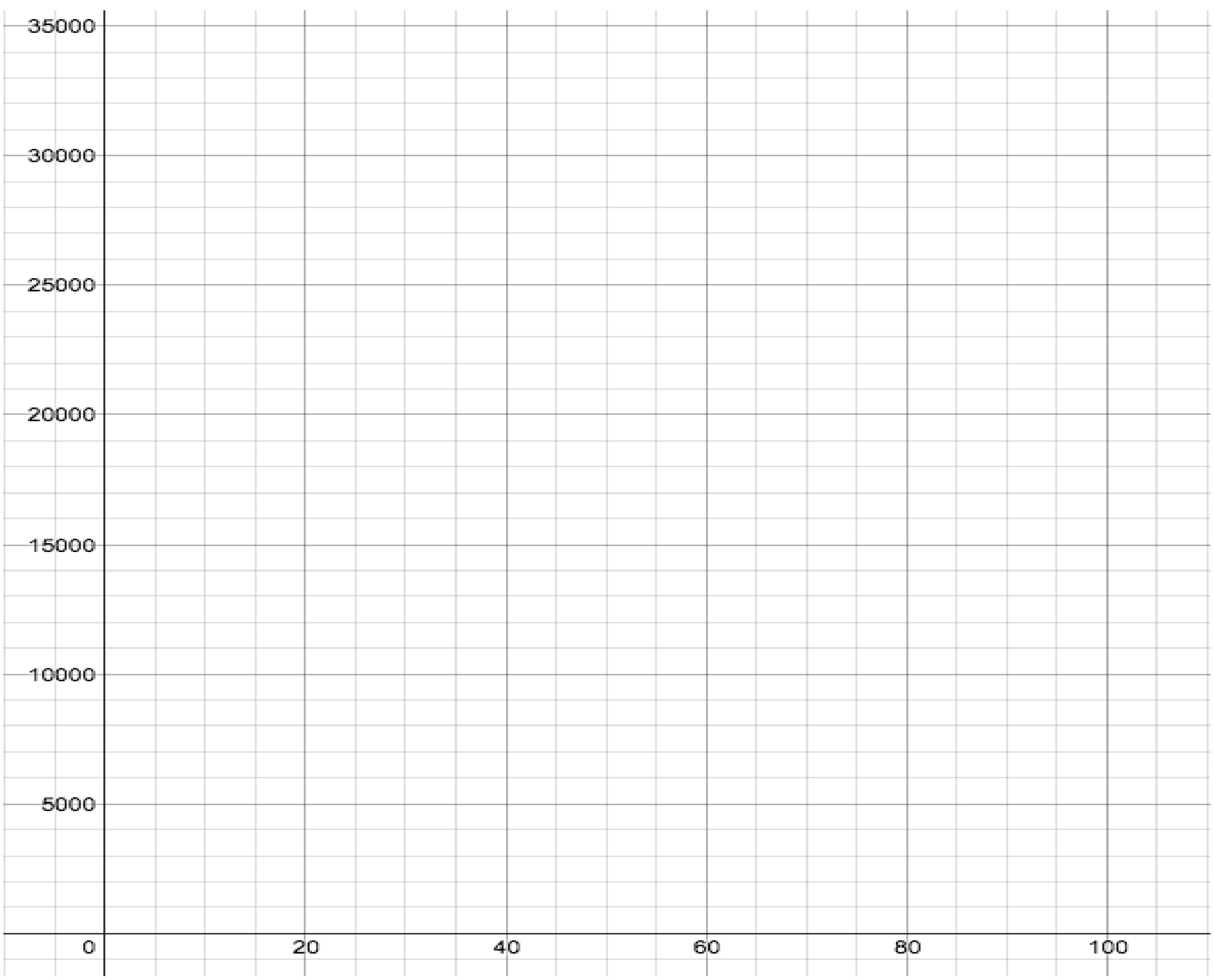
a. Equation for prize 1 (Linear Equation):

b. Equation for prize 2 (Exponential Equation):

c. Fill out the table to the right using your graphing calculator and each prize's equation.

d. Graph the results below

Week	Prize 1 Total	Prize 2 Total
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		



4. What are the exponential functions that match the growth of these numbers?

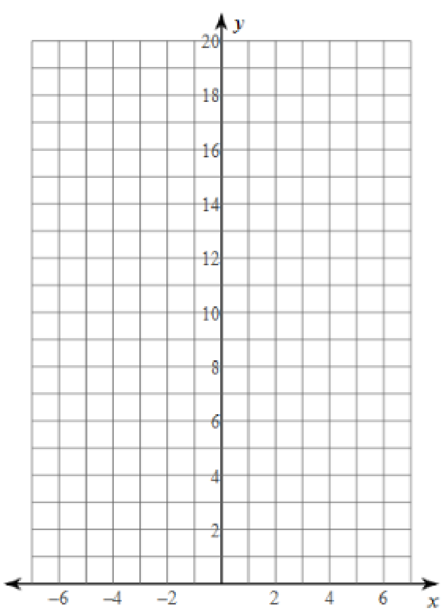
a) 3, 30, 300, 3000,....

b) 25, 5, 1, .2, …

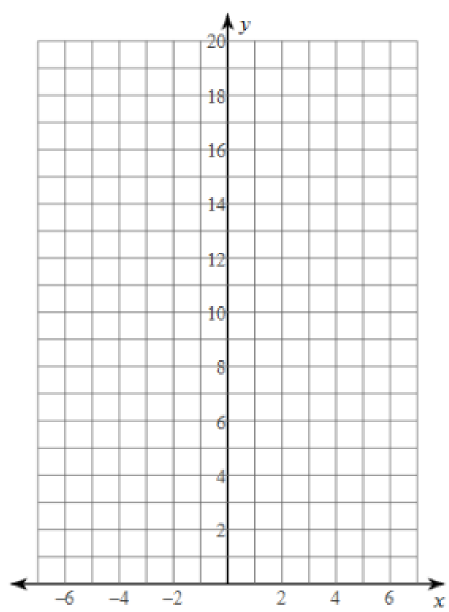
c) 9, 16.2, 29.16, 52.488, …

Sketch the graph of each function.

1) $f(x) = 5(2)^x$



2) $h(x) = 2(½)^x$



Algebra 1-2

Exponent Rules

Simplify. Your answer should contain only positive exponents.

1) $x^4y^2 \cdot xy^3$

2) $a^2 \cdot ba^4$

3) $(x^{-3}y^3)^4$

4) $(2x^3y^{-4})^2$

5) $\dfrac{2u^0v^4}{u^{-4}v^{-4}}$

6) $\dfrac{2a^4b^4}{a^2}$

7) $\left(\dfrac{2n^0}{2n^4 \cdot 2n^0}\right)^3$

8) $\dfrac{(2n^2)^{-1}}{2n^{-2} \cdot 2n^2}$

9) $\dfrac{(2x^2)^3}{x^2 \cdot x^{-3}x^0}$

10) $\left(\dfrac{p^{-1}}{2p^0 \cdot 2p}\right)^{-1}$

11) $\dfrac{(2m^4n^{-4} \cdot m^{-3}n^2)^2}{m^4n^{-2}}$

12) $\dfrac{(x^3)^4}{yx^3 \cdot 2yx^{-1}}$

13) $\dfrac{x^2 \cdot (2y^4)^{-3}}{2x^{-3}y^2}$

14) $\dfrac{x^{-2}y^{-2}}{(2x^4y^3)^4 \cdot (x^{-2}y^{-4})^0}$

15) $\dfrac{xy^2z^3}{(yz^{-4})^2 \cdot y^4z^2}$

16) $\dfrac{p^{-2}q^0 \cdot (q^{-2})^4}{2mq^4}$

17) $\dfrac{(2q^3r^{-3})^4}{2prq^0 \cdot rp^{-3}q^2 \cdot 2q^2r^{-4}}$

18) $\dfrac{(2yx^0z^{-2})^2 \cdot x^{-4}z^2}{2yz^0}$

Exponential Practice Test

Target A:
I can write, represent, evaluate and solve exponential functions using a table, graph, or situation.

1. Name the starting amount and the percentage gain or loss for each of the following equations.

	$f(x) = 5(.85)^x$	$y = 4(1.5)^x$	$g(x) = 19.4(1.152)^x$	$L(x) = 40(.16)^x$
Starting Value:	*5*			
Percentage Gain or Loss:	*15% loss*			

2. Behind you there is a sneeze. You don't know it at the time, but this begins a chain reaction that ripples through the school. It starts with only 3 students but the number infected grows every day at a rate of 44.7%.

a. What is the equation that models this situation?

b. How many students are infected on day 15?

c. About when is all of Wilson High School (1439 students) infected?

3. Fruit and vegetables begin losing nutrition the very second they are picked. Let's examine the amount of vitamin A in a single carrot. It starts at 46 grams and loses nearly 9% of its nutrition a day.

a. What is the equation that models this situation?

b. The recommended dose for vitamin A is 29 grams. Roughly, how many days would it take for a single carrot to no longer meet this requirement?

c. How many grams are there on day 20?

4. A single ticket to an Oregon home game averages 108 dollars. The demand for tickets has increased with all of their recent success, so ticket prices have grown at 11% for the past few years. Washington's home ticket price averages 150 dollars. Their ticket price is also increasing, but only at 4% since the demand is not as large.

a. Equation for Oregon's ticket price:

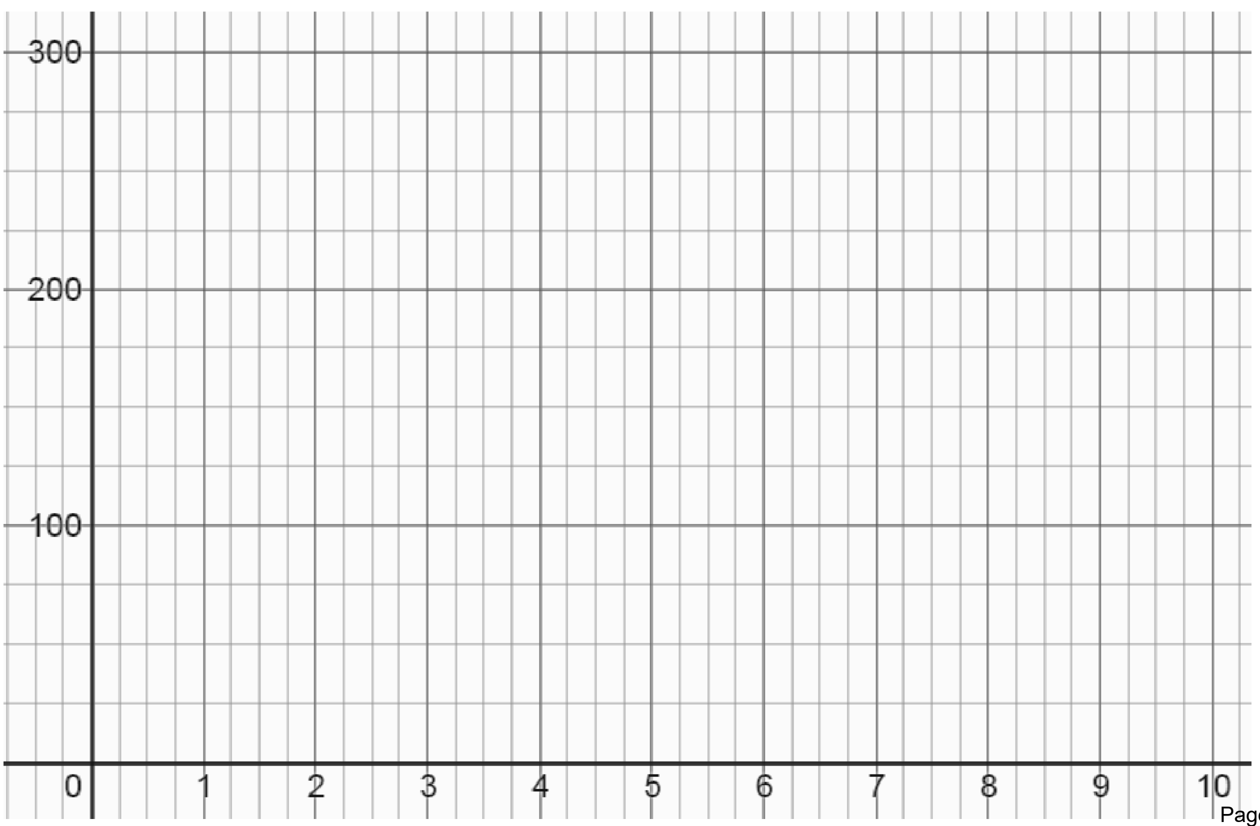
b. Equation for the Washington's ticket price:

c. Fill out the table to track the value of each ticket.

d. Graph the results.

e. When are the two home tickets approximately the same value?

Years from now	Oregon Ticket Price	Washington Ticket Price
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		



Target B:
I can write, represent, evaluate and solve exponential functions using a table, graph, or situation.

Simplify the following expressions. When you are done there should be no more parthesiss, negative exponents, or radicals.

5) $(x^6y^7)^2$

6) $(x^{-5}y^0)^3 \cdot (x^{15}z^4)^2$

7) $\dfrac{x^2y^{-5}}{x^4y^2}$

8) $\dfrac{15x^2y^5}{5x^4y^{200}}$

9) $\sqrt[4]{x^3}$

10) $\sqrt[7]{x^{10}}$

Target C:
I can write, represent, evaluate and solve exponential functions using a table, graph, or situation.

11) Find the equation that models these sets. Some are **linear** and some are **exponential**.

a. 3, 10, 17, 24, …

b. 5, 10, 20, 40, ….

c. 10, 8.5, 7, 5.5, …

d. 98, 19.6, 3.92, 0.784

e. The Hawaiian islands are sinking. Haleakala, on Maui, is 10,023 feet tall. Unfortunately, it is sinking 2 inches a year. (That's $\frac{1}{6}$ of a foot a year, bummer).

f. You take a stack of marshmallows with a volume of 15 cubic inches and start to microwave it. It then begins growing in the microwave at a rate of 40% every minute.

Below are several different algebraic expressions. Write them in expanded form, then unscramble them to find the hidden food name.

So	$\rightarrow$	$oi(lt)^2ra$
becomes	$\rightarrow$	$oi \cdot lt \cdot lt \cdot ra$
which is also	$\rightarrow$	$t \cdot o \cdot r \cdot t \cdot i \cdot l \cdot l \cdot a$

$huzn(ic)^2$

$ab(an)^2$

$$\frac{e(pa^2r)^2}{a^3r^2l^{-1}}$$

$$\frac{(m^0e^2c)^2k^2sh}{a^{-1}k}$$

$$\frac{(epr)^2}{(edp)^{-1}(jg^2z)^0}$$

$$\frac{p^0(om)^2u(r^3)^{\frac{1}{3}}}{(q^0s^4h)^{-1}s^3}$$

Exponential Challenge Problems

1. If you wanted an investment to be worth 50,000 in 50 years, how much would you have to originally invest (assuming you make the market average for the last 50 years of 7.0%)?

2. Pete was trying to figure out his finances. He knows that 7% is the average return of the stock market. His current plan is to invest $10,000 when he is 20 and he plans of retiring at 70. If want to retire with the same amount of money, but start investing when he is 45 what would he have to have as a starting value?

3. What is the exponential function that goes through the points (1, 6250) and (6,64)?

4. What takes longer to happen? 1 dollar turning into 10 dollars at 10% growth each year. Or 1,000,000 dollars turning into 10,000,000 dollars at 10% growth each year? Why?

5. Carbon dating looks at how much carbon-14 remains in a sample versus how much we would originally expect. Basically, half of the carbon-14 in a sample decays every 5730 years and since it decays at a known rate we can estimate how old a sample is. What is the equation in $y = ab^x$ form for a sample that originally has 20,000 grams of carbon-14. We know the original, but now we need to figure out the multiplier.

6. Solve for x: $4^{3x-2} = 1$

7. Solve for x: $9^{-3x} \cdot 9^{x} = 27$

8. Lindsey invested money in an account with a fixed interest rate. The interest is compounded annually. After 5 years her balance was $2741.69, and after 10 years her balance was $3416.53.
    a. Find an exponential equation that models these data.
    b. How much did Lindsey originally invest?
    C. What is the interest rate for the account?

Answer Key

Introduction into Exponential Functions

#1-2) Month 1: 2, Month 2: 4, Month 3: 8, Month 4: 16, … #3) $y = 1(2)^x$
#4a) Month 1: 2, Month 2: 12, Month 3: 72, Month 4: 432, … $y = 2(6)^x$
#4b) Month 1: 6, Month 2: 18, Month 3: 54, Month 4: 162, … $y = 6(3)^x$
#4c) Month 1: 4, Month 2: 16, Month 3: 64, Month 4: 256, … $y = 4(4)^x$

Exponential Decay

#1) Answers will differ #2) Within reason, it should remain consistent. There is a point where the ball is no longer accelerating when it drops; so the bounce for 3 miles high and 20 miles will be the same. #3) Answers will differ #4) Answers will differ

Exponential Situations PART 1

#1) 883.39	#2) $184,274.23	#3) $15.84	#4) $8203.82
#5) $2.69	#6) Between 22 and 23 years		#7) $9787.70

Exponential Situations PART 2

#1a) .97 #1b) 3% loss #1c) $y = 10(.97)^x$
#2a) $10,199.44 #2b) $9533.12 #3a) $y = 7300000000(1.015)^x$
#3b) Between year 6-7 year

Exponential Situations PART 3

#1b) 40 questions #2a) .975 so 2.5% loss #2b) Between 10 and 11 minutes
#2c) 9.6 degrees. You try drinking something that cold! #3a) $f(x) = 250x + 0$
#3b) $g(x) = 100(1.06)^x$ #4a) $y = 3*10^x$
#4b) $y = 25*(\frac{1}{5})^x$ #4c) $y = 9*(1.8)^x$

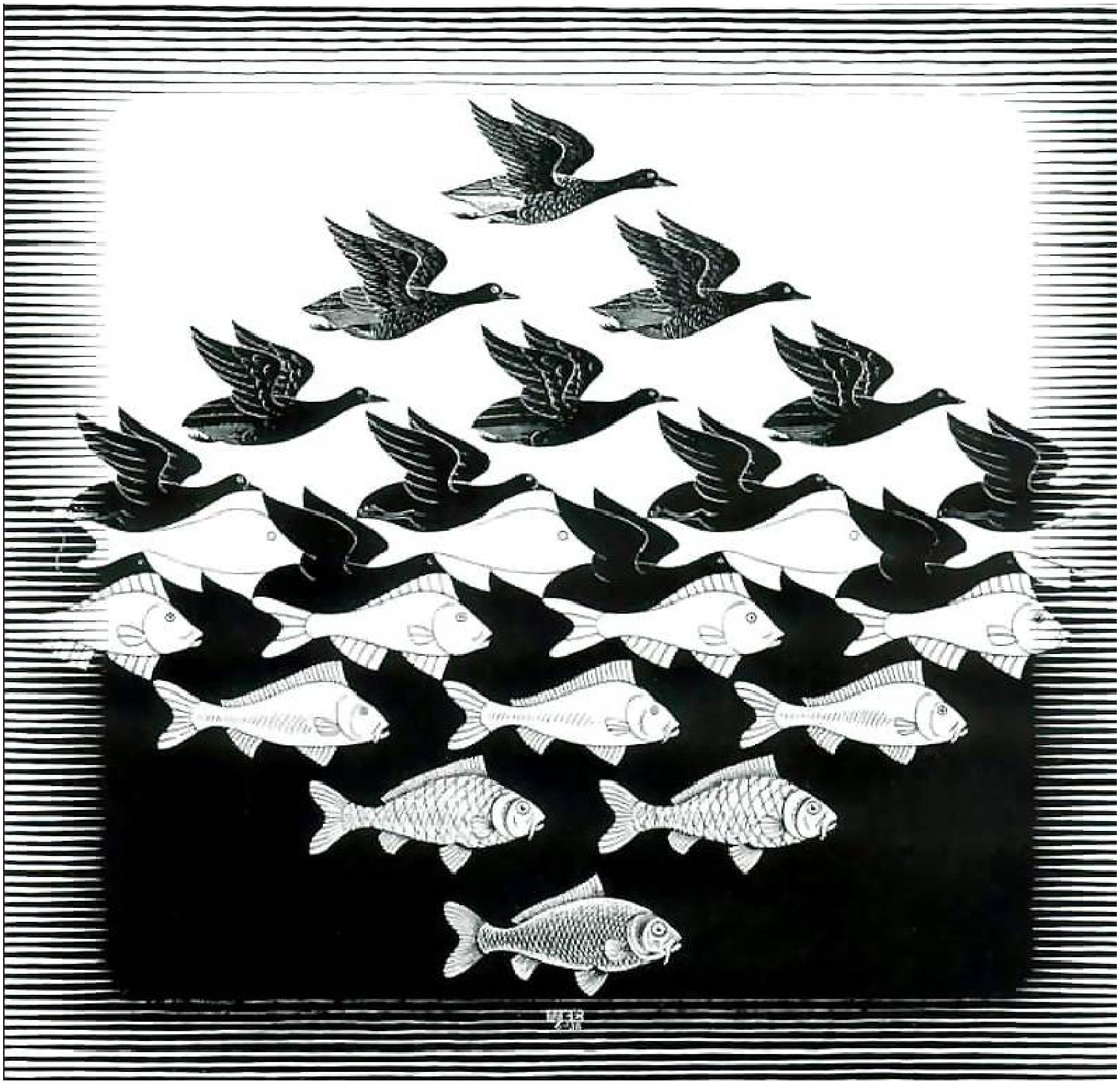
Exponent Rules

1) x^5y^5	2) a^6b	3) $\frac{y^{12}}{x^{12}}$	4) $\frac{4x^6}{y^8}$
5) $2u^4v^8$	6) $2a^2b^4$	7) $\frac{1}{8n^{12}}$	8) $\frac{1}{8n^2}$
9) $8x^7$	10) $4p^2$	11) $\frac{4}{n^2m^2}$	12) $\frac{x^{10}}{2y^2}$
13) $\frac{x^5}{16y^{14}}$	14) $\frac{1}{16x^{18}y^{14}}$	15) $\frac{z^9x}{y^4}$	16) $\frac{1}{2p^2q^{12}m}$
17) $\frac{4q^8p^2}{r^{10}}$	18) $\frac{2y}{z^2x^4}$		

Answer Key

Exponential Practice Test

#1) TOP ROW: 4, 19.4, 40 BOTTOM: 50% gain, 15.2% gain, 84% loss
#2a) $y = 3(1.447)^x$ #2b) 766 students #2c) Between 16-17 days
#3a) $y = 46(.91)^x$ #3b) Between 4-5 days #3c) 6.98 grams
#4a) $y = 108(1.11)^x$ #4b) $y = 150(1.04)^x$ #4e) Roughly 5 years
#5) $x^{12}y^{14}$ #6) $x^{15}z^8$ #7) $\frac{1}{x^2y^7}$ #8) $\frac{3}{x^2y^{195}}$
#9) $x^{3/4}$ #10) $x^{10/7}$ #11a) $y = 7x+3$ #11b) $y = 5(2)^x$
#11c) $y = -1.5x+10$ #11d) $y = 98(0.2)^x$ #11e) $y = -\frac{1}{6}x+10023$
#11f) $y = 15(1.4)^x$



Sky and Water
By: M.C. Escher